



EE 232 Lightwave Devices

Lecture 7: Absorption and Gain Coefficient, Bernard-Duraffourg Inversion Condition, Lineshape

Instructor: Ming C. Wu

University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.

EE232 Lecture 7-1

Prof. Ming Wu



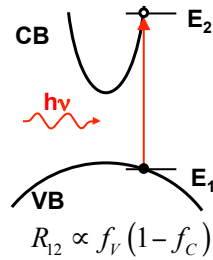
Absorption Coefficient

When CB and VB are partially filled:

Absorption Condition: VB is occupied, CB is empty

Absorption probability = $f_V(E_1)(1 - f_C(E_2))$

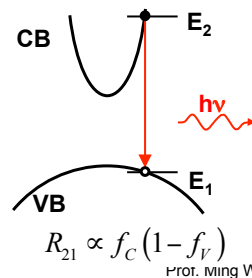
$$R_{12}(\hbar\omega) = \frac{2}{V} \sum_k \left[\frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_b - E_a - \hbar\omega) \right] f_V(1 - f_C)$$



Emission Condition: CB is occupied, VB is empty

Emission probability = $f_C(E_2)(1 - f_V(E_1))$

$$R_{21}(\hbar\omega) = \frac{2}{V} \sum_k \left[\frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_b - E_a - \hbar\omega) \right] f_C(1 - f_V)$$



EE232 Lecture 7-2

Prof. Ming Wu



Absorption Coefficient

Net absorption rate:

$$\begin{aligned}
 R(\hbar\omega) &= \frac{2}{V} \sum_k \left[\frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_b - E_a - \hbar\omega) \right] [f_V(1 - f_C) - f_C(1 - f_V)] \\
 &= \frac{2\pi}{\hbar} |H'_{ba}|^2 \int \frac{2d\vec{k}}{(2\pi)^3} \delta(E_g + \frac{\hbar^2 k^2}{2m_r^*} - \hbar\omega) [f_V(k) - f_C(k)] \\
 &= \frac{2\pi}{\hbar} |H'_{ba}|^2 \int \frac{2d\vec{k}}{(2\pi)^3} \delta(E_g + \frac{\hbar^2 k^2}{2m_r^*} - \hbar\omega) [f_V(k) - f_C(k)] \\
 &= \frac{2\pi}{\hbar} |H'_{ba}|^2 \rho_r(\hbar\omega - E_g) \left[f_V \left(-(\hbar\omega - E_g) \frac{m_r^*}{m_h^*} \right) - f_C \left(E_g + (\hbar\omega - E_g) \frac{m_r^*}{m_e^*} \right) \right]
 \end{aligned}$$

$$\alpha(\hbar\omega) = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \rho_r(\hbar\omega - E_g) [-f_g(\hbar\omega - E_g)]$$

Fermi Inversion Factor :

$$f_g(\hbar\omega - E_g) = f_C \left(E_g + (\hbar\omega - E_g) \frac{m_r^*}{m_e^*} \right) - f_V \left(-(\hbar\omega - E_g) \frac{m_r^*}{m_h^*} \right)$$

EE232 Lecture 7-3

Prof. Ming Wu



Fermi Inversion Factor

$$f_g(\hbar\omega - E_g) = f_C(E_2) - f_V(E_1)$$

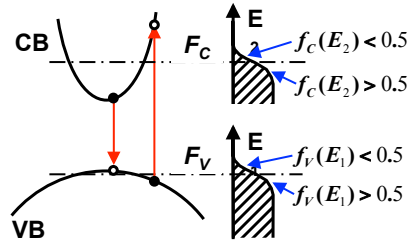
$$\begin{cases}
 E_2 = E_g + (\hbar\omega - E_g) \frac{m_r^*}{m_e^*} \\
 E_1 = -(\hbar\omega - E_g) \frac{m_r^*}{m_h^*}
 \end{cases}$$

And $k_1 = k_2$

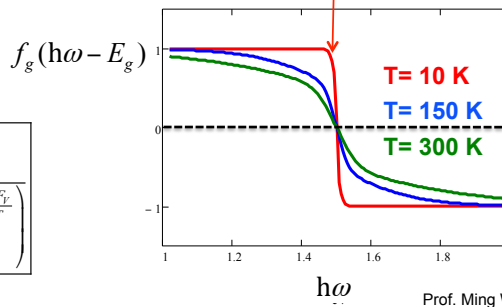
$$f_C(E_2) = \frac{1}{1 + e^{\frac{E_2 - F_C}{k_B T}}}$$

$$f_V(E_1) = \frac{1}{1 + e^{\frac{E_1 - F_V}{k_B T}}}$$

$$f_g(\hbar\omega - E_g) = \frac{e^{\frac{E_1 - F_V}{k_B T}} - e^{\frac{E_2 - F_C}{k_B T}}}{\left(1 + e^{\frac{E_2 - F_C}{k_B T}} \right) \left(1 + e^{\frac{E_1 - F_V}{k_B T}} \right)}$$



$$\Delta F = F_C - F_V$$



EE232 Lecture 7-4

Prof. Ming Wu



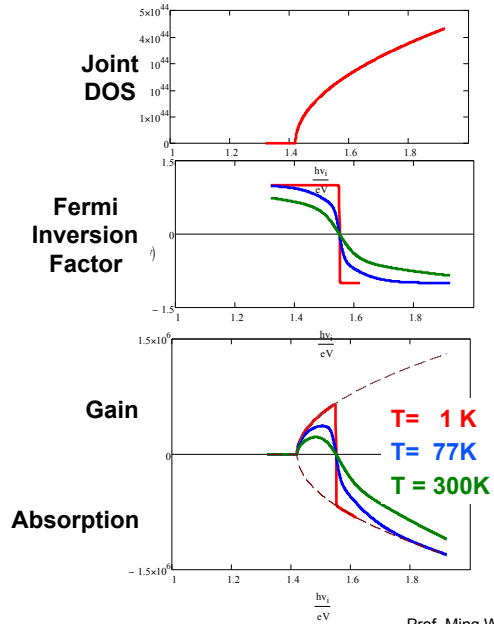
Optical Gain Coefficient

$$\alpha_0(\hbar\omega) = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \rho_r(\hbar\omega - E_g)$$

$$\alpha(\hbar\omega) = \alpha_0(\hbar\omega) \left[-f_g(\hbar\omega - E_g) \right]$$

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$

$$g(\hbar\omega) = \alpha_0(\hbar\omega) f_g(\hbar\omega - E_g)$$



EE232 Lecture 7-5

Prof. Ming Wu



Optical Gain Coefficient versus Bias

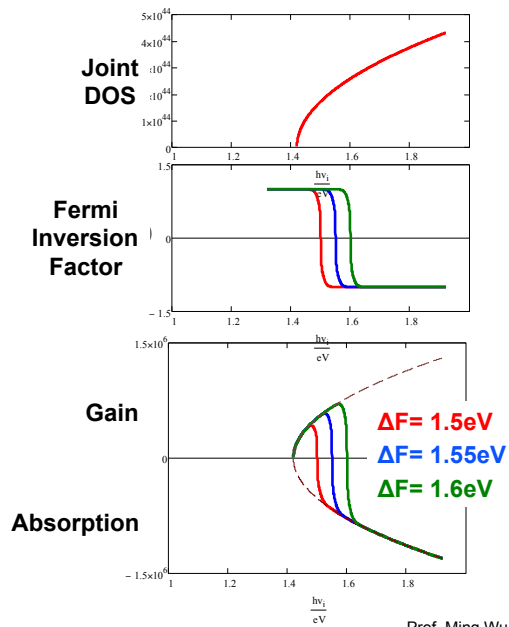
$$g(\hbar\omega) = \alpha_0(\hbar\omega) f_g(\hbar\omega - E_g)$$

Bernard-Duraffourg
Inversion Condition

$$\Delta F = F_C - F_V > E_g$$

Spectral Range of Gain

$$E_g < \hbar\omega < \Delta F = F_C - F_V$$



EE232 Lecture 7-6

Prof. Ming Wu



Lineshape Broadening

$$g(\hbar\omega) = C_0 \left| \hat{e} \cdot \overline{P_{cv}} \right|^2 \int dE \rho_r(E) \delta(E = \hbar\omega - E_g) f_g(E)$$

Lineshape broadening due to intraband scattering of electrons

→ Replace delta function by a Lorentzian lineshape function:

$$\delta(E = \hbar\omega - E_g) \rightarrow L(E - (\hbar\omega - E_g)) = \frac{1}{\pi} \frac{\hbar / \tau_{in}}{(E - (\hbar\omega - E_g))^2 + (\hbar / \tau_{in})^2}$$

$$\text{FWHM} = \Gamma = \frac{2\hbar}{\tau_{in}}$$

τ_{in} : intraband relaxation time $\sim 0.1 \text{ ps}$

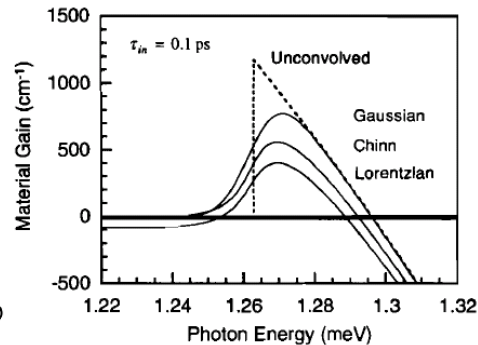
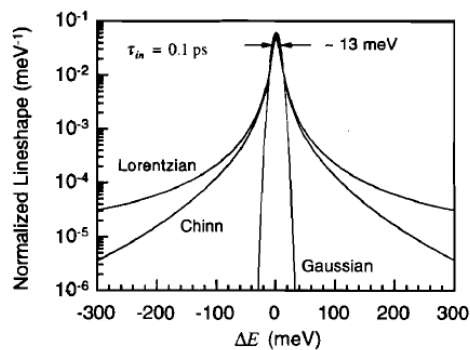
$$g(\hbar\omega) = C_0 \left| \hat{e} \cdot \overline{P_{cv}} \right|^2 \int dE \rho_r(E) \left[\frac{1}{\pi} \frac{\hbar / \tau_{in}}{(E - (\hbar\omega - E_g))^2 + (\hbar / \tau_{in})^2} \right] f_g(E)$$

EE232 Lecture 7-7

Prof. Ming Wu



Lineshape Functions and Gain Spectra



Source: Coldren & Corzine, p.133

EE232 Lecture 7-8

Prof. Ming Wu